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A new model for trinucleons

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Abstract. The binding energy, the bare form factor and the photodisintegration, bremsstrahlung-weighted and integrated cross sections of the trinucleons (^{3}H and ^{3}He) have been calculated earlier by using a modified Feshbach wave function and a velocity-dependent potential. We recalculate these properties of trinucleons with a modified Irving wave function but using the same velocity-dependent potential. We now find better agreement with experiments. This shows that our present model for the trinucleons is much more satisfactory than the earlier model using a modified Feshbach wave function.

1. Introduction

Some years ago Srivastava calculated the binding energy, the photodisintegration integrated cross section ($\sigma_{\text{int}} = \int_0^\infty \sigma(W)\mathrm{d}W$) and bremsstrahlung-weighted cross section ($\sigma_b = \int_0^\infty (\sigma/W)\,\mathrm{d}W$) and the bare form factor $F_B(q^2)$ of the trinucleons (^{3}H and ^{3}He) using a velocity-dependent potential and three-parameter modified Feshbach wave function (Srivastava 1964, 1965 a, b, Levinger and Srivastava 1965). It was found that the results of these calculations did not agree very well with the experiments. Subsequently, some workers (Tang and Herndon 1965) have pointed out that the modified Feshbach wave function may not have enough flexibility to describe properly the eigenfunctions of the three-nucleon systems. In view of these remarks it seems worth while to examine whether using some other wave function to describe the three-body system leads to better agreement with experiments. In the present paper we make such an investigation with a modified Irving wave function (Irving 1951).

2. Variational calculation of the binding energy of the trinucleons

We assume the nuclear forces to be central, spin independent, but velocity dependent and, therefore, the ground-state wave function of the triton to be completely symmetric in the relative spatial coordinates of the three nucleons. Hence, we choose as our trial function for the triton the Irving wave function ψ_1 (Irving 1951) and its suitable modifications ψ_2 and ψ_3:

$$\psi_1 = \frac{N_1 \exp\{-\alpha(\Sigma_{i<j} r_{ij}^2)^{1/2}\}}{(\Sigma_{i<j} r_{ij}^2)^n} \tag{1}$$

$$\psi_2 = \frac{N_2[\exp\{-\alpha(\Sigma_{i<j} r_{ij}^2)^{1/2}\} - \exp\{-\lambda(\Sigma_{i<j} r_{ij}^2)^{1/2}\}]}{(\Sigma_{i<j} r_{ij}^2)^n} \tag{2}$$

$$\psi_3 = \frac{N_3[\exp\{-\alpha(\Sigma_{i<j} r_{ij}^2)^{1/2}\} + A\exp\{-\lambda(\Sigma_{i<j} r_{ij}^2)^{1/2}\}]}{(\Sigma_{i<j} r_{ij}^2)^n}. \tag{3}$$

Here $i, j = 1, 2$ and 3, the N are the appropriate normalizing constants and α, λ, A and n are the variational parameters.

Under appropriate assumptions (Srivastava 1965 a), the zeroth-order wave function for ^{3}He is the same as the wave function ψ for the triton (^{3}H). The binding energy of ^{3}He then differs from that of the triton by the Coulomb energy of interaction of protons, E_c (^{3}He). The first-order perturbation theory gives

$$E_c(^3\text{He}) = e^2 \int |\psi|^2 \frac{1}{r_{23}} \mathrm{d}\tau \tag{4}$$

where we have assumed nucleons 2 and 3 in ^{3}He are protons.

The effective two-body nuclear potential is the average of potentials in the ^1S and ^3S states and is the same as used earlier (Srivastava 1965 a)

$$V_{\text{eff}}(r_{ij}) = -\tfrac{1}{2}(1+X_{\text{stat}})(V_0)_{\text{stat}}\exp\left(-\frac{2r_{ij}}{\beta_s}\right) + \frac{(V_0)_{\text{vel}}}{2M}\{p^2\omega_s(r_{ij})+\omega_s(r_{ij})p^2\} + \frac{X_{\text{vel}}(V_0)_{\text{vel}}}{2M}\{p^2\omega_t(r_{ij})+\omega_t(r_{ij})p^2\} \quad (5)$$

where

$$\omega(r_{ij}) = \exp\left(-\frac{2r_{ij}}{\beta'}\right). \quad (6)$$

(Subscripts s and t denote singlet and triplet states respectively.) The ^1S part of the potential fits the singlet effective range and the ^1S and ^1D phase shifts up to 340 MeV laboratory energy of the incident proton in p–p scattering (Rojo 1961) while the ^3S part fits the binding energy of the deuteron and Breit's ^3S phase shifts at $E_{\text{lab}} = 147$, 270 and 310 MeV (Srivastava 1965 a).

The values of the potential parameters are (Srivastava 1965 a)

$$(V_0)_{\text{stat}} = 100\,\text{MeV}, \quad (V_0)_{\text{vel}} = 2$$
$$X_{\text{stat}} = 1{\cdot}84, \quad X_{\text{vel}} = 0{\cdot}55 \quad (7)$$
$$1/\beta_s = 0{\cdot}625\,\text{fm}^{-1}, \quad 1/\beta_s' = 1{\cdot}4\,\text{fm}^{-1}, \quad 1/\beta_t' = 1\,\text{fm}^{-1}.$$

Using the transformations and method of integration given by Irving (1951) we calculate

$$E^3_H = \langle T\rangle^3_H + \langle V_{\text{stat}}\rangle^3_H + \langle V_{\text{vel}}\rangle^3_H \quad (8)$$

for each of the trial functions given by equations (1), (2) and (3) and minimize it with respect to the variational parameters. Next we calculate E_c (^{3}He) by evaluating the integral given by equation (4). Table 1 summarizes the results of these calculations.

Table 1. Variational calculation of the binding energy of trinucleons

Trial function		Binding energy (MeV)	Coulomb energy (MeV)	Best values of the parameters in the trial function α (fm^{-1})	λ (fm^{-1})	A	n
(*a*)	ψ_1	6·8459	0·6591	0·55			0
	ψ_2	8·1845	0·7235	0·69	1·16	−1	0
	ψ_3	8·2167	0·7243	0·70	1·23	−1·20	0
(*b*)	ψ_1	6·08	0·597	0·58			
	ψ_2	7·08	0·660	0·771	1·105	−1	
	ψ_3	7·17	0·663	0·732	1·415	−1·305	
Experimental		8·49	0·764				

(*a*) Present calculation

(*b*) Srivastava (1965 a). Using modified Feshbach wave function, i.e.

$$\psi_3 = N_3[\exp\{-\tfrac{1}{2}\alpha(r_{12}+r_{13}+r_{23})\} + A\exp\{-\tfrac{1}{2}\lambda(r_{12}+r_{13}+r_{23})\}]$$

ψ_2 and ψ_1 may be obtained by replacing A by -1 and 0 respectively in the above equation.

The value of the binding energy of the triton obtained for the trial function given by equations (1), (2) and (3) suggests that convergence has almost been obtained. That the highest value of the binding energy is obtained with the four-parameter trial function is

in accord with the theory of variational calculation. Also, the agreement of the calculated values of the binding energy of the triton and the Coulomb energy of ^{3}He with the experimental values is much better than that obtained earlier (Srivastava 1965 a).

3. Calculation of σ_{int}, σ_b and $F_B(q^2)$ for the trinucleons

In this section we use the triton ground-state wave function ψ_3 (henceforth we shall call it ψ_0), given by equation (3) with the best values of the parameters given in table 1 and the nuclear potential given by equations (5)–(7), to calculate the photodisintegration integrated (σ_{int}) and bremsstrahlung-weighted (σ_b) cross sections and the bare form factor of the trinucleons.

We follow the analysis of Rustgi (1957) and Srivastava (1965 b) in calculating σ_{int} and σ_b. The expressions for these quantities for our modified Irving wave function ψ_0 are given by

$$\begin{aligned}\sigma_{\text{int}} = \frac{4\pi^2 e^2 \hbar}{3Mc}\Bigg(&1+\frac{56M}{\pi\hbar^2}(1+X_{\text{stat}})(V_0)_{\text{stat}}(x+\tfrac{1}{2}y)\\&\times F\{\alpha^{-8}I(C_{s,\alpha})+2A\left(\frac{\alpha+\lambda}{2}\right)^{-8}I(C_{s,(\alpha+\lambda)/2})+A^2\lambda^{-8}I(C_{s,\lambda})\}\\&+\frac{32}{3\pi}(V_0)_{\text{vel}}F[\{\alpha^{-6}K(C'_{s,\alpha})+2A\left(\frac{\alpha+\lambda}{2}\right)^{-6}K(C'_{s,(\alpha+\lambda)/2})\\&+A^2\lambda^{-6}K(C'_{s,\lambda})\}\\&+X_{\text{vel}}\{\alpha^{-6}K(C'_{t,\alpha})+2A\left(\frac{\alpha+\lambda}{2}\right)^{-6}K(C'_{t,(\alpha+\lambda)/2})\\&+A^2\lambda^{-6}K(C'_{t,\lambda})\}]\Bigg)\end{aligned} \tag{9}$$

where x and y are the fractions of the Majorana and Heisenberg exchange forces respectively,

$$F=\left\{\alpha^{-6}+2A\left(\frac{\alpha+\lambda}{2}\right)^{-6}+A^2\lambda^{-6}\right\}^{-1}$$

$$C_{s,\alpha}=\left(\frac{2}{3}\right)^{1/2}\frac{1}{\alpha\beta_s},\qquad C'_{s,\alpha}=\left(\frac{2}{3}\right)^{1/2}\frac{1}{\alpha\beta_s'},\qquad C'_{t,\alpha}=\left(\frac{2}{3}\right)^{1/2}\frac{1}{\alpha\beta_t'},\ \text{etc.}$$

$$\begin{aligned}I(P)&=\int_0^{\pi/2}\frac{\sin^4\theta\cos^2\theta\,d\theta}{(1+P\cos\theta)^8}\\&=\frac{16P^4+16P^2+1}{16(1-P^2)^{13/2}}\cos^{-1}P\\&\quad-\frac{P(919+2346P^2+216P^4-16P^6)}{1680(1-P^2)^6}\qquad\text{for } P<1\end{aligned}$$

$$\begin{aligned}K(Q)&=\int_0^{\pi/2}\frac{\sin^2\theta\cos^2\theta\,d\theta}{(1+Q\sin\theta)^6}\\&=\frac{1}{120}\left\{\frac{15(1+6Q^2)\cos^{-1}Q}{(1-Q^2)^{9/2}}-\frac{Q(81+28Q^2-4Q^4)}{(1-Q^2)^4}\right\}\qquad\text{for } Q<1\end{aligned}$$

$$\langle r^2\rangle_{00}=\frac{7F}{6}\left\{\alpha^{-8}+2A\left(\frac{\alpha+\lambda}{2}\right)^{-8}+A^2\lambda^{-8}\right\} \tag{10}$$

and

$$\sigma_b=\frac{4\pi^2}{3}\frac{e^2}{\hbar c}\langle r^2\rangle_{00}. \tag{11}$$

On substituting the values of the parameter of the wave function ψ_0 and the potential parameters (cf. equation (7)) we obtain the values of σ_{int}, $\langle r^2\rangle_{00}$ and σ_b. The results are shown in table 2.

Table 2. Photodisintegration cross section of trinucleons

	σ_{int} (MeV mbn)	$\langle r^2\rangle_{00}^{1/2}$ (fm)	σ_b (mbn)
Present calculation	$\sigma_{\text{int}} = \frac{4\pi^2 e^2\hbar}{3Mc}\{1{\cdot}044 + 0{\cdot}809(x+\frac{1}{2}y)\}$	1·68	2·72
	$= 58{\cdot}0$ for $x+\frac{1}{2}y = 0{\cdot}5$†		
	$= 67{\cdot}7$ for $x+\frac{1}{2}y = 0{\cdot}8$‡		
Srivastava's calculation[a]	$\sigma_{\text{int}} = \frac{4\pi^2 e^2\hbar}{3Mc}\{1{\cdot}038 + 0{\cdot}724(x+\frac{1}{2}y)\}$	1·92	3·5
	$= 56{\cdot}0$ for $x+\frac{1}{2}y = 0{\cdot}5$†		
	$= 64{\cdot}7$ for $x+\frac{1}{2}y = 0{\cdot}8$‡		
Experimental[b]	62 ± 6	1·64	$2{\cdot}53 \pm 0{\cdot}19$

† Serber mixture.
‡ Rosenfeld or Inglis mixture.
a, Srivastava (1965 b); using modified Feshbach wave function.
b, Fetisov *et al.* (1965).

We calculate the bare form factor (Srivastava 1964, Schiff 1964) by the expression

$$F_B(q^2) = \int |\psi_0|^2 \exp(i\mathbf{q}\,.\,\mathbf{r})\, d^3\mathbf{r}_i \tag{12}$$

and following the analysis of Schiff (1964) we obtain

$$F_B(q^2) = F[G(\alpha, q^2) + 2AG\{\tfrac{1}{2}(\alpha+\lambda), q^2\} + A^2 G(\lambda, q^2)] \tag{13}$$

where

$$G(x, q^2) = x^{-6}\left(1 + \frac{q^2}{18x^2}\right)^{-7/2}.$$

The experimental values of the bare form factor are computed from the expression (Srivastava 1964, Schiff 1964)

$$F_B(q^2) = \frac{2F_E(^3\text{He}) + F_E(^3\text{H})}{3(F_{Ep} + F_{En})}. \tag{14}$$

In equation (14) we use the results of Collard *et al.* (1965) for the charge form factor of ^{3}He and ^{3}H obtained from electron–trinucleon scattering experiments while the values of $F_{Ep} + E_{En}$ are taken from Dudelzak's thesis (Jain and Srivastava 1967). These theoretical and experimental values of $F_B(q^2)$ for the trinucleons are given in figure 1. In figure 1 we have also given the values of $F_B(q^2)$ obtained by using a hard-core potential (Tang and Herndon 1965).

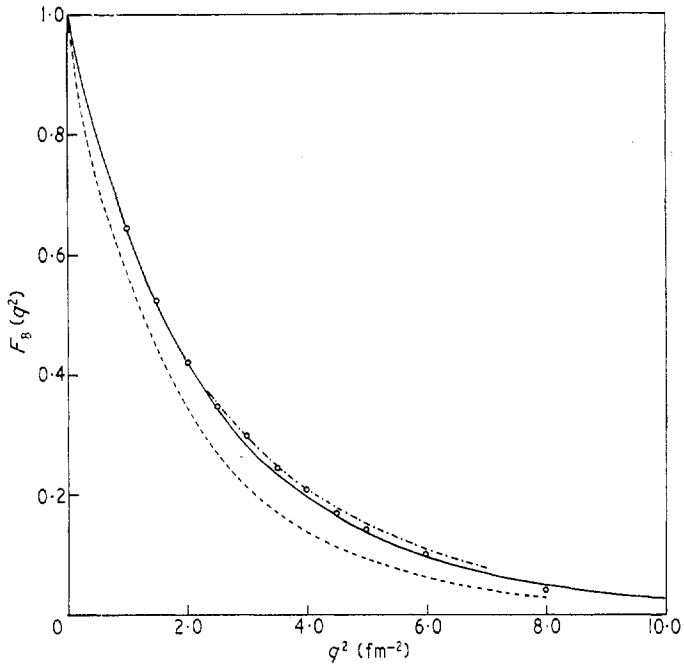

Figure 1. Bare form factor of trinucleons plotted against the momentum transferred squared, q^2. The full curve represents the present calculation while the broken curve and chain curve show the results of Levinger and Srivastava (1965) and Tang and Herndon (1965), respectively. The circles correspond to the experimental results.

4. Conclusion

A comparison of our present values for the binding energy, photodisintegration cross section and the bare form factor of the trinucleons with the experimental values shows that the results of the present calculations are in excellent agreement with experiments. Also, this agreement is much better than that obtained by using a modified Feshbach wave function (Srivastava 1965 a, b, Levinger and Srivastava 1965). Thus we find that the trinucleons are described much better by a modified Irving wave function than by a modified Feshbach wave function.

As we have mentioned earlier in this paper, our two-body central velocity-dependent potential gives a good fit to low- and high-energy scattering data and the binding energy of the deuteron (Rojo 1961, Srivastava 1965 a). It also gives the values of the binding energy (Jain and Srivastava 1968), integrated and bremsstrahlung-weighted photodisintegration cross sections (Srivastava and Jain 1967) and the charge form factor (Jain and Srivastava 1967) of ^{4}He in reasonable agreement with those obtained from experiments. All this and our present calculation show that our two-body central velocity-dependent potential (cf. equations (5)–(7)) is also quite good.

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